C.—The patient's serum, 3 vols.; staphylococcus suspension, 1 vol.; A. E. W.'s washed corpuscles, 3 vols.

Tube 1.—Phagocytic power (bacteria in 15 P.W.B.C. counted and averaged)	....	30
Tube 2.— Do. do.		26

"The Magnetic Expansion of some of the less Magnetic Metals." By P. E. SHAW, B.A., D.Sc. Communicated by Professor J. H. POYNTING, F.R.S. Received May 22,—Read June 18, 1903.

1. Abundant research has been made on the magnetic expansion of iron, nickel, and cobalt, notably as regards the exact relation between field (H) and expansion per unit length ($\delta l/l$), by S. Bidwell* and H. Nagaoka.† Bismuth also has been investigated by Bidwell, C. G. Knott, Van Aubel, and A. P. Wills. But there seems to be no recorded research on any materials other than the four mentioned.

Outside the ferro-magnetic group bismuth has the largest susceptibility (k) of any substance; and the tacit assumption seems to have been made that if bismuth shows no change in length as the field varies, it is vain to look for it in less susceptible metals.

But in the case of the ferro-magnetics there is no direct relation between k and $\delta l/l$. Thus, iron has maximum susceptibility six times as much as nickel, and yet expands far less for any known field.

Again, cobalt has maximum susceptibility one-eighth of that of iron, yet expands about as much.

There being, therefore, no close relation between susceptibility and magnetic expansion, it seems possible that there may be appreciable movement for large fields in the case of metals outside the ferro-magnetic group. This paper gives an account of tests applied to bismuth, silver, aluminium, copper, zinc, brass, bronze, lead, and tin. (Not much importance should be attached to the results for lead and tin owing to the softness of these metals; they tend to work loose in their fittings at each end.) The work has taken from first to last nearly two years: the specimens have been repeatedly changed and the magnetic and measuring parts of the apparatus modified in various ways. In this way searching tests have been applied to the investigation.

For a long time it appeared (1) that all these metals contracted, the contraction being roughly proportional to the field, (2) that all the metals showed permanent magnetisation, on the hysteresis principle,

* 'Phil. Trans.,' A, 1888.

† 'Phil. Mag.,' Jan., 1894.

just as was shown by Nagaoka* for iron and nickel. But step by step these inferences have been proved to be fallacious, as greater care was taken in the exclusion of iron from the apparatus and in the exact setting of the rod symmetrical in the coil and free from mechanical connection with it. The final conclusion reached is that no true expansion, positive or negative, can be detected within the limits of the experiment.

2. The magnetic and length-measuring apparatus have been described in previous communications† and cannot easily be summarised intelligibly.

I am indebted to Mr. Schott (see the Note appended) for working out the value of the reduction factor κ in the expression

$$H = 4\pi N\gamma \,.\, \kappa.$$

The ordinary expression $H = 4\pi N\gamma$ requires modifying for such a short solenoid as the one used by me. The numerical calculation of the integrals in the note gives for κ the value 0·867.

The rods are cylinders, 19 cm. long, 6 mm. diameter (except bismuth and lead 12 mm. diameter).

The coil is 19 cm. long and has 3604 turns, hence $N = 190$.

Using the values of currents (γ) below, we derive the corresponding value of field (H), where H is the effective uniform field as expressed above.

γ (in ampères)...	0·80	1·45	2·00	2·50	3·10	3·80	4·43	5·30	6·40	7·70	9·25.
H (c.g.s. units) ..	164	295	413	510	635	784	900	1090	1313	1588	1906.

These currents were read on an ammeter which was periodically calibrated. They were always applied gradually by using a sliding liquid resistance, vanishing to *nil*.

The coil was made somewhat short. This was done in order, with a given resistance in the windings, to concentrate the field on the rod.

Again, the rod occupied the whole length of the coil. It is customary when experimenting on a ferro-magnetic to build the core up of three parts, the ferro-magnetic occupying the central third of the length (where the field is practically uniform), whilst the two end thirds of the length are of brass. Here the assumption is made that brass itself undergoes no expansion. I could not proceed on this plan, for we have no certainty that any metal is neutral, *i.e.*, is quite free from magnetic expansion. So I decided to have the rod the same length as the coil and trust to calculation to obtain the mean effective

* *Loc. cit.*

† Shaw and Laws, 'Electrician,' Feb., 1901, and Feb., 1902.

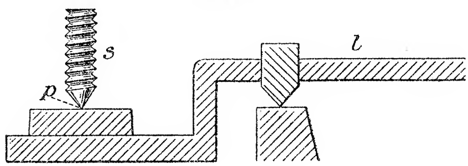
field throughout its length. In the present research it is immaterial if there be a small error in the calculated value of field; but it is essential that any expansion produced should be due to the specimen exclusively.

As regards the rod, we are not troubled by the demagnetising factor (N) in the well-known expression for effective field,*

$$H' = H - NI,$$

where I = magnetic intensity and $H = 4\pi n\gamma$. In the present instance, since k is very small for all the metals in question, I becomes insensible.

FIG. 1.



The electric micrometer was calibrated by using a simple electric micrometer† in the following way, which is accurate enough for present purposes. Suppose l is the last lever of the system of levers and p is the measuring point; we require to know how many divisions of the electric micrometer screw correspond to a known movement of p. Remove the coil and accessories which are above p, and bring screw s of the simple micrometer down to p. When electric contact is completed take a reading of the graduated head of the screw s. Rotate s so that the surfaces at p are separated by a known amount, say 4 microns. Then find out how much the screw of the electric micrometer has to be rotated to bring the surfaces together again and produce electric contact once more.

As a means of four results, 4 microns movement at p corresponds to 14,900 divisions on the electric micrometer screw, hence—

1 micrometer division corresponds to $2{\cdot}68 \times 10^{-8}$ cm., which (since the rod is 19 cm. long) is equivalent to $1{\cdot}4 \times 10^{-9}$ of the length of the rod.

Allowing for uncertainties in readings, I should say that three divisions on the scale represent the smallest difference that can be detected with certainty.

3. As seen above, the range of field was considerable and the upper limit high (1900 c.g.s.), whilst the measuring apparatus was very delicate.

* Ewing's 'Magnetic Induction,' 2nd edition, p. 24.

† 'Phys. Rev.,' March, 1903.

Various strains, other than those for which we are looking, will be liable to occur, and to show themselves in the measuring apparatus, *e.g.*, due to (1) solenoidal suction of the coil on the end; (2) directive action of the earth's field on the magnetised coil; (3) attractive action on the coil of any iron in its neighbourhood.

It is very difficult to eliminate all such false strains. At first I worked with the coil attached to the lower end of the rod, counterpoising the coil to remove all strain from the rod. This method was employed by S. Bidwell,* it obviates solenoidal suction and may on that account be useful with a ferro-magnetic core, but in my experience it was troublesome. I then placed the coil on a stand separate from the core and its supports, taking care to centre the rod in the coil. The coil and core must not come into contact or the small movements of the former will displace the latter.

A great deal of time was spent in finding out the best conditions of working; thousands of readings often apparently quite consistent for a long time were taken down. These readings varied from 1×10^{-6} cm. to 1×10^{-8} cm. But gradually, as the elimination of disturbing causes proceeded, the result in the case of every metal tried was found to be *nil.*

Bismuth was tested with special interest on account of the work already done on it.

S. Bidwell† recorded a small expansion, but later‡ he reported that although the field used was 1500 c.g.s., and the unit of measurement 1/70,000,000 of the length of the rod, yet no movement could be found.

No other attempt seems to have been nearly as searching as this last, although Knott,§ Van Aubel,|| and lately A. P. Wills¶ have also taken up the matter and record the same negative result.

After working with one specimen of bismuth, I had the material recast of somewhat less diameter, taking care to remove the surface, which might obtain impurities from the loam used in the casting. In both field and length measurement, the test which I have applied to bismuth appears to go further than any previous one, for, as shown above, the maximum field is 1900 c.g.s., and the smallest movement per unit length readable is $\delta l/l = 4 \times 10^{-9}$ cm.

The metals used were obtained specially pure from Messrs. Johnson and Matthey; they were analysed for traces of iron by the sulpho-cyanide colour test. The following list shows the amount of iron in 10,000 parts of metal: bismuth, 0·88; silver, 1·0; tin, 0·1; aluminium, 72; zinc, 0·68; lead, 0·26; copper, 4·8; bronze, 7·0; brass, 4·0.

4.—(*a*) It has generally been supposed that a small trace of iron in

* *Loc. cit.*

† 'Phil. Trans.,' A, 1888.

‡ 'Nature,' July, 1899.

§ 'Nature,' June, 1899.

|| 'Nature,' Aug., 1899.

¶ 'Phys. Rev.,' July, 1902.

a metal would produce magnetic expansion on its own account which would mask any small expansion due to the metal. From the analyses above, we may test this hypothesis.

Aluminium, silver, bronze, copper and brass all have more iron than 1 in 10,000 parts. Now in a field of 1900 c.g.s., the value of $\delta l/l$ for iron is about $-4 . 10^{-5}$; assuming simple superposition of the expansions of the two ingredients, then 1 of iron in 10,000 of metal should produce a value for $\delta l/l$ equal to -4×10^{-9}.

But, as shown above, this movement is readable on the electric micrometer. In aluminium, and even in brass and copper, the impurity should have produced quite a large movement in my apparatus.

It would be interesting to examine the relation of the amount of iron in a specimen to the magnetic expansion. My experiments certainly do not confirm the simple superposition theory.

(*b*) At one time it was not uncommon for writers on magnetic expansion to deduct a contraction $e = B^2/8\pi M$ from the whole expansion observed, this being supposed to be always caused by the Maxwell stress in the magnetised substance.

But this course has been shown by Dr. Chree to be unjustifiable.*

In these experiments, putting $B = H = 1900$ c.g.s. and giving M the value 10^{11} c.g.s., we have $e = 2 \times 10^{-6}$ cm. for each unit length of specimen, or for the rods used (19 cm. long) $e = 4 \times 10^{-5}$ cm., roughly.

Any such quantity would be easily detected by the apparatus. Hence the conclusion is that the stress does not act wholly, if at all, on the matter. If only one ten-thousandth of the stress acted directly on the matter the effect would have been perceptible.

The negative results of these experiments, like those of Sir O. Lodge,† seem to show that the mechanical connection between matter and ether, if existent, is inappreciable by our present methods.

I have pleasure in acknowledging my obligation to the Royal Society for the Government grant made to me in 1901, by which the cost of this research has been defrayed.

* See letters by Chree and others, 'Nature,' Jan., 1896.

† 'Phil. Trans.,' A, 1897.

Note by G. A. Schott, B.A., B.Sc.

Calculation of the Elongation, due to the Magnetic Field, of the Cylindrical Bars.

The dimensions of Dr. Shaw's apparatus are as follows :—

Length of solenoid and bar	$(2l)$	$=$ 19 cm.
Mean radius of solenoid	(a)	$=$ 2·75 cm.
Depth of winding	$(2c)$	$=$ 2·40 cm.
Number of layers........................	$(2p)$	$=$ 22
,, turns per layer	(n)	$=$ 164
Distance between layers	(h)	$=$ 4/35 cm.
Radius of bar	(b)	$=$ from 0·3 to 0·6 cm.

Use cylindrical co-ordinates (z, ρ) where O z is the axis of the bar and the origin is at the centre.

Let the axial and radial components of the magnetic force (H), due to the solenoid, be Z, R, and those due to the induced magnetisation H′, Z′, R′.

R R′ are of order ρ, and inside the bar are small compared with Z; they give terms in the strain of order ρ^2, which are less than 1 per cent. of the whole and may be neglected. In the same way there are terms of order ρ^2 in Z, which can be neglected. Thus we may for our purpose regard the magnetic force inside the bar as axial and constant over any cross section, but variable from end to end.

The intensity of magnetisation is given by $I = k\,(H+H')$. The volume density of magnetism is div. $I = (H+H')$ div. $k + k$ div. H′, since div. $H = 0$.

For all the metals used by Dr. Shaw, k is exceedingly small (at most 10^{-5}), and its square may be neglected. Div. k involves the differential coefficients of k with respect to the strains as well as space-variations of the strains. The former, though they may be large compared to k, yet are small absolutely; the latter also are small; hence, div. k is negligible. Since H′ is of order I, the volume density of magnetism is of order k^2 and negligible. The induced magnetism is confined to the surface of the bar, and the force H′, being of order k, may be neglected.

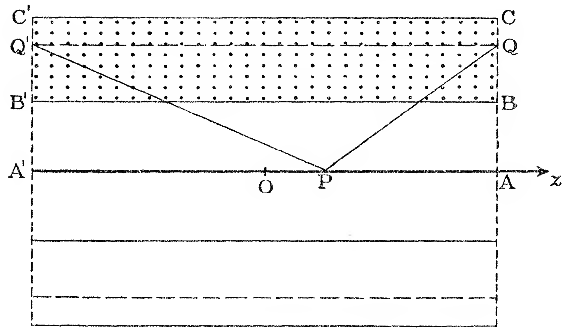
Thus we need only take account of the component Z.

We shall require the mean values of Z and Z^2, say $\overline{Z}$ and $\overline{Z^2}$.

The figure gives a meridian section. A′A is the axis, BCC′B′ the section of the upper half of the solenoid, QQ′ the section of a layer of wire of radius r.

As we neglect variations of Z across the section of the bar, we need only consider its value at a point P (z) on the axis.

FIG. 2.



By a well-known formula the magnetic force (ΔZ), due to the layer QQ' carrying current γ, is given by

$$\Delta Z = \frac{\pi n\gamma}{l}\left[\frac{l-z}{\sqrt{[(l-z)^2+r^2]}}+\frac{l+z}{\sqrt{[(l+z)^2+r^2]}}\right].$$

Here

$$r = a-c+qh, \qquad q = 0, 1, \ldots 20, 21.$$

Maclaurin's summation formula gives

$$\Sigma f(r) = \frac{2c+h}{h}\left\{f(a)+\frac{c(c+h)}{6}f''(a)\right\} = 22\,\{f(a)+0{\cdot}184c^2f''(a)\},$$

where terms of order $\frac{h^2}{6}f''(a) = \frac{c^2}{660}f''(a)$ have been neglected.

The total number of ampère turns in the solenoid is $22n\gamma$, and the number per centimetre is $22n\gamma/2l$; let the magnetic force inside an infinitely long solenoid with the same number of ampère turns per centimetre be H, then $H = \frac{4\pi\, 22n\gamma}{2l} = 4\pi N\gamma$.

Maclaurin's formula gives for the actual magnetic force

$$Z = \Sigma\Delta Z = \tfrac{1}{2}H\left[\frac{l-z}{\sqrt{[(l-z)^2+a^2]}}\left\{1-0{\cdot}184\frac{c^2}{(l-z)^2+a^2}+0{\cdot}552\frac{a^2c^2}{\{(l-z)^2+a^2\}^2}\right\}\right.$$
$$\left.+\frac{l+z}{\sqrt{[(l+z)^2+a^2]}}\left\{1-0{\cdot}184\frac{c^2}{(l+z)^2+a^2}+0{\cdot}552\frac{a^2c^2}{\{(l+z)^2+a^2\}^2}\right\}\right] \quad \ldots\ldots \quad (1).$$

Now we have $2l = 19$, $a = 2{\cdot}75$, $c = 1{\cdot}20$, so that

$$\frac{a}{l} = 0{\cdot}290, \qquad \frac{a}{\sqrt{(4l^2+a^2)}} = 0{\cdot}143, \qquad \frac{c^2}{a^2} = 0{\cdot}192.$$

Also

$$\frac{1}{2l}\int_{-l}^{l}\frac{(l-z)\,dz}{\{(l-z)^2+a^2\}^{n+}} = \frac{1}{2l}\int_{-l}^{l}\frac{(l+z)\,dz}{\{(l+z)^2+a^2\}^{n+\frac{1}{2}}}$$

$$= \frac{a}{2l}\frac{1}{(2n-1)\,a^{2n}}\left\{1-\left(\frac{a}{\sqrt{(4l^2+a^2)}}\right)^{2n-1}\right\}.$$

Hence from (1)

$$\overline{Z} = \mathrm{H}\,(0{\cdot}866 - 0{\cdot}184\times 0{\cdot}192\times 0{\cdot}145\times 0{\cdot}857 + 0{\cdot}552\times 0{\cdot}192\times 0{\cdot}145 \times 0{\cdot}332 = 0{\cdot}867\,.\,\mathrm{H}.$$

Again, (1) gives, after simple algebraic transformations,

$$\begin{aligned} Z^2 = \mathrm{H}^2\Big[\,& 0{\cdot}500 - 0{\cdot}268\frac{a^2}{(l-z)^2+a^2} + 0{\cdot}071\frac{a^4}{\{(l-z)^2+a^2\}^2} - 0{\cdot}024\frac{a^6}{\{(l-z)^2+a^2\}^3} \\ & - 0{\cdot}268\frac{a^2}{(l+z)^2+a^2} + 0{\cdot}071\frac{a^4}{\{(l+z)^2+a^2\}^2} - 0{\cdot}024\frac{a^6}{\{(l+z)^2+a^2\}^3} \\ & + \frac{l^2-z^2}{\sqrt{[(l-z)^2+a^2]}\sqrt{[(l+z)^2+a^2]}} \\ & \left\{0{\cdot}500 - 0{\cdot}018\frac{a^2}{(l-z)^2+a^2} + 0{\cdot}053\frac{a^4}{\{(l-z)^2+a^2\}^2} - 0{\cdot}018\frac{a^2}{(l+z)^2+a^2} + 0{\cdot}053\frac{a^4}{\{(l+z)^2+a^2\}^2}\right\}\Big], \end{aligned}$$

where terms in c^4 have been neglected.

$\overline{Z^2}$ involves the following integrals :—

$$\frac{1}{2l}\int_{-l}^{l}\left[\frac{dz}{\{(l-z)^2+a^2\}^n} + \frac{dz}{\{(l+z)^2+a^2\}^n}\right],$$

and

$$\frac{1}{2l}\int_{-l}^{l}\left[\frac{1}{\{(l-z)^2+a^2\}^n} + \frac{1}{\{(l+z)^2+a^2\}^n}\right]\frac{(l^2-z^2)\,dz}{\sqrt{[(l-z)^2+a^2]}\sqrt{[(l+z)^2+a^2]}}.$$

Call them I_n, J_n.

In I_n put $l\pm z = a\cot\theta$; then

$$\mathrm{I}_n = \frac{a}{l}\frac{1}{a^{2n}}\int_{\tan^{-1}a/2l}^{\frac{1}{2}\pi}\sin^{2n-2}\theta\,.\,d\theta$$

$$= \frac{0{\cdot}290}{a^{2n}}\left[\frac{1\,.\,3\,.\,.\,.\,(2n-3)}{2\,.\,4\,.\,.\,.\,(2n-2)}\frac{\pi}{2} - \frac{1}{2n-1}\left(\frac{a}{2l}\right)^{2n-1}\right],\ \text{approximately.}$$

Thus

$$I_1 = \frac{0{\cdot}414}{a^2}, \qquad I_2 = \frac{0{\cdot}228}{a^4}, \qquad I_3 = \frac{0{\cdot}171}{a^6}.$$

In J_n put $z = \sqrt{(l^2+a^2)}\,\dfrac{\cos(\theta+\alpha)}{\cos(\theta-\alpha)}$, where α is defined by $\tan 2\alpha = \dfrac{a}{l}$; also let $t = \tan\alpha$.

With the above values of a/l we have $\alpha = 8^\circ\, 5'$, $t = 0{\cdot}142$.

Clearly t^3 is negligible except where it is divided by $\cos\theta$ or $\sin\theta$, which become of order t at one or other limit. This substitution gives

$$J_n = \frac{1}{a^{2n}}\frac{2t}{(1+t^2)^{n-1}}\int_\alpha^{\frac{1}{2}\pi-\alpha}(1+t^{2n}\sec^{2n}\theta)(\cos\theta+t\sin\theta)^{2n-2}(\sin\theta-t\cos\theta)\,d\theta$$

$$= \frac{1}{a^{2n}}\frac{2t\sqrt{(1+t^2)}}{2n-1}\Big[1+t^{2n}.\,(1+t^2)^n - \{1+(1+t^2)^n\}\Big(\frac{2t}{1+t^2}\Big)^{2n-1}$$

$$+\frac{2nt^{2n}}{(1+t^2)^{2n-1/2}}\int_\alpha^{\frac{1}{2}\pi-\alpha}(1+t\tan\theta)^{2n-1}.\,\sec\theta\tan\theta\,d\theta\Big],$$

whence

$$J_0 = 1{\cdot}484, \quad J_1 = \frac{0{\cdot}242}{a^2}, \quad J_2 = \frac{0{\cdot}097}{a^4}\ \ldots$$

Hence we get

$$\overline{Z^2} = H^2\,[0{\cdot}500 - 0{\cdot}268\,.\,a^2I_1 + 0{\cdot}071\,.\,a^4I_2 - 0{\cdot}024a^6I_3$$
$$+ 0{\cdot}250\,.\,J_0 - 0{\cdot}018\,.\,a^2J_1 + 0{\cdot}053\,.\,a^4J_2]$$

$$= 0{\cdot}77\,.\,H^2.$$

Lastly, (1) gives the values at the ends of the bar, say Z_l and Z_l^2. Resuming, we get the following values :—

$$\overline{Z} = 0{\cdot}867H, \qquad Z_l = 0{\cdot}495H,$$
$$\overline{Z^2} = 0{\cdot}77H^2, \qquad Z_l^2 = 0{\cdot}245H^2.$$